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CONTROL CONSIDERATIONS FOR OPTIMUM POWER PROPORTIONMENT IN TURBINE-PROPELLER ENGINES

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SUMMARY

A turbine-propeller engine operating at constant speed was analyzed in order to provide information relative to (1) factors influencing optimum proportionment of power between propeller and jet, (2) improvement in engine performance attainable from controlled proportionment of power, and (3) control relations for optimum proportionment of power.

The conditions for optimum proportionment of power were found to vary with propeller characteristics for each propeller installation on a given engine. With the proper choice of a propeller, the necessity for controlled proportionment of power is minimized. When a propeller resulting in inefficient engine performance with fixed power proportionment is used, however, substantial improvement can be obtained with controlled proportionment. For the general case of an engine with varying propeller and turbine efficiencies, a two- or three-position exhaust-nozzle area can provide adequate optimum power proportionment. The theoretical expression for a completely variable area is too complex to be directly applicable to a practical control system.

INTRODUCTION

In previous investigations it has been indicated that the proportionment of power between the propeller and the jet of a turbine-propeller engine affects power output and it has also been shown that the thrust output attains an optimum value dependent on the proportionment of power (reference 1). This optimum is the maximum net thrust obtainable for a given amount of power available at the turbine inlet. Whether the gains to be derived from optimum-power proportionment are commensurate with the complexity of the control apparatus required remains to be evaluated.

The gains and the complexity of optimum-power-proportionment control have therefore been evaluated at the NACA Lewis laboratory. In this analysis, the proportionment of power is assumed to be controlled by variations in the area of the exhaust nozzle. A method by which the advantages of controllable exhaust-nozzle area may be ascertained is presented and illustrated and the attendant control problems are discussed. A theoretical analysis of optimum power proportionment is evolved in which the optimum division of power between the propeller and the jet is shown to be a function of propeller and turbine characteristics. In order to evaluate the necessity for optimum power proportionment, typical engine and propeller data are used to calculate the effect of variation in exhaust-nozzle area on engine performance.

The engine performance was calculated for a through-flow engine having a sea-level take-off rating of approximately 2400 horsepower including the effect of jet thrust. The significant engine components were a 14-stage axial-flow compressor with a maximum pressure ratio of 6, a single-stage turbine, and an 11.35:1 planetary reduction gear for reducing propeller-shaft speed. At a constant engine speed of 13,000 rpm, compressor-blade-tip velocity was 936 feet per second and the turbine-pitch-line velocity was 1475 feet per second. The turbine-annulus-nozzle area was 30 square inches. A schematic diagram of the turbine-propeller engine is shown in figure 1.

Results of the analysis are presented in terms of the effect of change in exhaust-nozzle area on maximum engine thrust over a range of flight speeds and altitudes, including sea-level take-off conditions. The desirability of controlled proportionment of power as dependent on propeller characteristics is discussed. Application of the theoretical equation as a control parameter is also discussed. The analysis and the conclusions drawn are valid only for a turbine-propeller engine that operates at constant engine speed over its entire power range, with critical flow existing at the turbine nozzles.

THEORETICAL BASIS FOR OPTIMUM-POWER PROPORTIONMENT

In the following theoretical discussion, the relation between the power inputs to the propeller and to the jet of a turbine-propeller engine is derived in terms of optimum jet velocity, which may be regulated by control of the exhaust-nozzle area.

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For any altitude, ram, engine-speed, and fuel-flow conditions, the following approximate equation based on the temperature-entropy diagram of the thermodynamic engine cycle (fig. 2) and the symbol notation of appendix A can be written:

$$\frac{hp_t}{\eta_t} + \frac{W_g V_j^2}{2g550\eta_j} = K \tag{1}$$

The net thrust can be expressed as

$$F_n = F_p + \frac{W_g}{g} V_j - \frac{W_a}{g} V_0 \tag{2}$$

It is assumed that η_j does not vary with turbine back pressure. Differentiating equations (1) and (2) with respect to hp_p and setting $\frac{\partial F_n}{\partial hp_p} = 0$ for maximum thrust at a given fuel flow (which corresponds to the condition for optimum efficiency) yield

$$\left.\begin{aligned} \frac{1}{\eta_t}\frac{\partial hp_t}{\partial hp_p} - \frac{hp_t}{\eta_t^2}\frac{hp_t}{\partial hp_p} + \frac{W_g V_j}{550g\eta_j}\frac{\partial V_j}{\partial hp_p} &= 0 \\ 0 = \frac{\partial F_p}{\partial hp_p} + \frac{W_g}{g}\frac{\partial V_j}{\partial hp_p} & \end{aligned}\right\} \tag{3}$$

Elimination of $\frac{\partial V_j}{\partial hp_p}$ from equation (3) gives

$$V_j = \frac{550\eta_j}{\eta_t \frac{\partial F_p}{\partial hp_p}}\left(\frac{\partial hp_t}{\partial hp_p} - \frac{hp_t}{\eta_t}\frac{\partial \eta_t}{\partial hp_p}\right) \tag{4}$$

If gear losses are neglected, $hp_t = hp_p + hp_c$; because hp_c is constant for the assumed conditions, $\frac{\partial hp_t}{\partial hp_p} = 1$. Therefore

$$V_j = \frac{550\eta_j}{\eta_t \frac{\partial F_p}{\partial hp_p}} \left(1 - \frac{hp_t}{\eta_t} \frac{\partial \eta_t}{\partial hp_t}\right) \tag{5}$$

Equation (5) may be expressed in terms of propeller efficiency. The expression for F_p is 1023

$$F_p = \frac{550\eta_p \, hp_p}{V_0}$$

and

$$\frac{\partial F_p}{\partial hp_p} = \frac{550\eta_p}{V_0} = \left(1 + \frac{hp_p}{\eta_p} \frac{\partial \eta_p}{\partial hp_p}\right)$$

so that equation (5) becomes

$$V_j = \frac{\eta_j}{\eta_t \eta_p} V_0 \frac{\left(1 - \frac{hp_t}{\eta_t} \frac{\partial \eta_t}{\partial hp_t}\right)}{\left(1 + \frac{hp_p}{\eta_p} \frac{\partial \eta_p}{\partial hp_p}\right)} \tag{6}$$

When changes in jet efficiency are neglected, the optimum jet velocity becomes a function only of turbine characteristics, propeller characteristics, and flight velocity. For different propeller installations on the same engine, the optimum jet velocity theoretically varies directly with propeller characteristics.

CALCULATION OF ENGINE PERFORMANCE WITH CONTROLLED POWER PROPORTIONMENT

For the assumed conditions of critical flow in the turbine nozzle and constant values of altitude, flight velocity, turbine-inlet temperature, and engine speed, all other engine variables upstream of the turbine are constant, regardless of the proportions of available power delivered to the propeller and to the jet. The power available at the turbine inlet is therefore unaffected by changes in the power output of the turbine and the jet, so that the effect of variations in the division of power between propeller and jet on the net engine output can be calculated for the fixed condition upstream of the turbine.

The proportioning of power was varied by assuming various values of turbine-power output. Values of turbine output were assumed, ranging from the minimum required to drive only the compressor to the maximum obtainable from the power available at the turbine inlet. For the assumed value of turbine output, the power delivered to the propeller and the thrust output of the propeller were directly obtained from the excess shaft horsepower (turbine output less compressor, reduction-gearing, and accessory requirements) and from the propeller characteristics.

The required exhaust-nozzle area was determined from calculated turbine-outlet pressure and temperature, and from the known gas flow through the engine. With the area known, the jet thrust at 100-percent nozzle efficiency could be calculated. Finally, from the sum of propeller and jet thrust less intake-air drag, the value of net thrust was obtained corresponding to the calculated exhaust-nozzle area. The same method was used for different turbine-inlet temperatures at the same altitude and velocity, and the entire process repeated for each altitude and velocity assumed. From the relation between net thrust and exhaust-nozzle area, the area corresponding to peak thrust can be obtained by inspection. Calculations were made for sea-level take-off velocities of 0, 50, and 100 miles per hour at an engine temperature ratio of 4.5 and for velocities of 200, 300, 400, and 500 miles per hour for engine temperature ratios from 2.75 to 5.0 at altitudes of sea level and 35,000 feet.

Inasmuch as the theoretical discussion indicates that optimum proportionment of power is dependent on propeller characteristics, the described operating conditions were evaluated for two propellers of different diameters. Both propellers are represented in figure 3. The first evaluation was made for a propeller 13.5 feet in diameter, which is capable of absorbing maximum engine power under all operating conditions. The second evaluation was made for a propeller 10 feet in diameter. This propeller performed efficiently over only a limited range of flight conditions and was approximately the smallest propeller that could be used.

A detailed description of the methods of calculation is given in appendix B.

RESULTS AND DISCUSSION

The results of the calculations are shown in figures 4 and 5. In these curves, proportionment of power is represented by exhaust-nozzle area with an increasing area indicating an increasing amount of propeller power. The lines of constant engine temperature ratio T_4/T_2 represent constant available power at the turbine inlet. 1023

Figure 4 shows the variation of effective exhaust-nozzle area with net thrust of the engine at various engine temperature ratios T_4/T_2 for a propeller with a diameter of 13.5 feet. The figure consists of curves for (a) sea-level take-off, (b) flight velocities at sea level, and (c) flight velocities at an altitude of 35,000 feet. Sections of heavy lines on the curves of figure 4 indicate the range of exhaust-nozzle areas permissible for the attainment of thrust values within 2 percent of the peak attainable thrust. In addition, a line is shown that approximates the optimum fixed exhaust-nozzle area over the range of flight velocities investigated. These additions to figure 4 were obtained from inspection and provide a visual picture of the increase in thrust available from optimum-power proportionment in contrast with the thrust attainable with the best possible fixed exhaust-nozzle area. Whenever optimum-thrust conditions did not occur at an exhaust-nozzle area less than 200 square inches, the deviations were calculated from the thrust obtained at this arbitrary maximum area.

The performance obtained with a propeller 10 feet in diameter is presented in figure 5, which is similar to figure 4. The engine performance shown in figures 4 and 5 indicates that, for all flight conditions and turbine-inlet temperatures considered, an optimum net thrust is either attained or approached within the range of exhaust-nozzle areas investigated.

Although a drag curve of an airframe has not been utilized, the performance curves of figures 4 and 5 apply to both steady-state and transient thrust requirements. For a given installation, steady-state operation at a specific velocity and altitude would require only one value of thrust, depending upon the drag of the airframe. This value of thrust would determine single values of turbine-inlet temperature and exhaust-nozzle area for optimum proportionment of power. Transient operation, however, requires values of thrust deviating from the steady-state requirement, so that a range of turbine-inlet temperatures must be considered. The performance curves presented indicate the thrust obtained by an instantaneous change in engine temperature ratio without an accompanying change in engine speed or flight velocity.

Comparison of the line representing operation with a fixed exhaust nozzle having an effective area of 170 square inches, which appears to be the best selection if constant area is used, and the 2-percent deviations in maximum net thrust in figure 4 indicates that, when a propeller 13.5 feet in diameter is utilized, relatively small increases in thrust can be gained by varying the area of the exhaust nozzle. An exhaust-nozzle area of 170 square inches would in almost all cases result in a thrust output that is within 2 percent of the peak value. At the sea-level take-off conditions shown in figure 4(a), a sacrifice in thrust of approximately only 2 percent results from use of this area. There appears to be a trend toward a loss in thrust with increasing velocity at very low values of engine temperature ratio with a fixed area of 170 square inches. A combination of low temperature and high velocity represents a condition of deceleration, and the inefficient use of available power would be relatively unimportant. It is therefore indicated that for the engine investigated a propeller can be so selected that control of exhaust-nozzle area is unnecessary.

The performance of the engine with a propeller 10 feet in diameter (fig. 5) differs considerably from that obtained with the larger propeller. For all flight conditions considered, the best fixed exhaust-nozzle area appears to be approximately 140 square inches. Using a fixed area of this value seriously limits the thrust output at the sea-level take-off conditions, as shown in figure 5(a). At the take-off conditions shown, the thrust peaks at exhaust-nozzle areas ranging between approximately 65 and 75 square inches. The thrust falls off considerably beyond this range of areas. Use of a fixed area of 140 square inches also results in limitations in the acceleration characteristics at low flight velocities. This limitation is evident at a flight velocity of 200 miles per hour for both sea level and 35,000 feet (figs. 5(b) and 5(c)). Steady-state flight at a velocity of 200 miles per hour requires a relatively low value of turbine-inlet temperature. Acceleration from this condition at the fixed exhaust-nozzle area by increasing the turbine-inlet temperature results in a value of thrust considerably less than the peak value. For an altitude of 35,000 feet, an increase in the engine temperature ratio from a value of 4.5 to 5.0 actually reduces the thrust output of the engine. These results for the 10-foot propeller indicate that a variable exhaust nozzle is desirable for improvement of engine performance.

The difference in engine performance with the 13.5-foot propeller and with the 10-foot propeller is attributed to the relative efficiencies of the two propellers. The 13.5-foot propeller is capable of absorbing increasing engine horsepower with improved propeller efficiency. The efficiency of the 10-foot propeller begins to decrease before absorbing maximum engine power. Although the 10-foot propeller may be considered somewhat undersize for the engine investigated, a small propeller could have application in conjunction with a light-weight engine. For such an application, proportionment of power becomes a critical factor in determining engine performance. The use of propeller characteristics differing considerably from the conventional characteristics used in this analysis may, in some cases, alter these general conclusions.

The dependence of exhaust-nozzle-area control requirements on propeller characteristics verifies the results of the theoretical analysis. The theoretical analysis showed that jet velocity at optimum proportionment of power for a given engine varies only with the characteristics of the propeller used.

As indicated, the possibility exists that control of the exhaust-nozzle area may be desirable for a given installation. In order to be correct theoretically, the exhaust-nozzle area would be controlled according to equation (6). This equation states that jet velocity must be varied according to a function of the propeller and turbine characteristics for each flight velocity, with jet efficiency assumed constant. Equation (6) in the form shown is too complex to be applicable directly to a practical control system. It may have utility, however, when turbine efficiency is essentially constant. If a turbine has this characteristic, jet velocity at optimum proportionment of power is a function only of propeller characteristics and flight velocity. Under conditions of constant flight velocity and altitude, a relation between propeller horsepower and propeller efficiency is defined and subsequently determines a relation between jet velocity and propeller horsepower. Optimum proportionment of power could be obtained under all operating conditions with a variable-area exhaust nozzle so actuated as to adjust the jet velocity to a value determined by measured values of propeller horsepower, flight velocity, and altitude.

If the propeller and turbine efficiencies undergo only small changes over the operating range, the control of the exhaust-nozzle area would not generally be required; but, if engine performance indicates that it is desirable, a control system could function to maintain constant the ratio of jet velocity to flight velocity. With the assumption of representative values of efficiency, this ratio is about 1.5 or 1.6.

Although optimum proportionment of power is a function of propeller and turbine characteristics, inefficient operation with a constant-area exhaust nozzle may occur only over a limited region of the operating range. This characteristic is exhibited by the performance of the engine with the 10-foot propeller at take-off and acceleration at low flight velocities. For this particular engine, approximately the full advantage of a completely variable exhaust-nozzle area could be obtained with a selection of two or three fixed areas. The smallest area would be used for take-off, an intermediate area for acceleration, if desired, and the largest area for cruise and high-speed flight. The area may be automatically set by a control sensing the flight condition or manually set by the pilot. This type of control system is more desirable from a practical standpoint with regard to the complexity of the control relation and the mechanical difficulties of a completely variable area.

It is to be noted that specific application of exhaust-nozzle-area control may be limited as dictated by the limiting conditions of the compressor, the turbine, and the burner operation. Such limitations have not been considered in this general analysis.

CONCLUSIONS

From an analysis of a turbine-propeller engine operating at constant speed with critical flow existing at the turbine nozzles, the following conclusions may be drawn:

1. Gains to be derived from optimum-power proportionment vary with propeller characteristics for each propeller installation on a given engine.

2. The necessity of controlled proportionment of power can be minimized through selection of a propeller having the proper characteristics.

3. Utilization of available energy can be improved by control of proportionment of power if design specifications require a propeller that cannot efficiently absorb engine power under all flight conditions.

4. For the general case of an engine with varying propeller and turbine efficiencies, a two- or three-position area control can

provide adequate optimum-power proportionment, whereas the theoretical expression for a completely variable area is too complex for practical control purposes.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
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APPENDIX A

SYMBOLS

The following symbols are used in this investigation of a turbine-propeller engine:

A_e effective area of exhaust nozzle, sq in.

C_P propeller power coefficient $\left[\dfrac{0.0005\ hp_p}{\sigma\left(\frac{N_p}{1000}\right)^3\left(\frac{d}{10}\right)^5}\right]$

c_p specific heat at constant pressure, Btu/(lb)(°R)

c_v specific heat at constant volume, Btu/(lb)(°R)

D intake-air drag, lb

d propeller diameter, ft

F thrust, lb

g mass ratio, 32.2 lb mass/slug

hp horsepower

J mechanical equivalent of heat, 778 ft-lb/Btu

K constant

N speed, rpm

n speed, rps

P total pressure, lb/sq in.

p static pressure, lb/sq in.

R gas constant, 53.3 ft-lb/(lb)(°R)

T total temperature, °R

t static temperature, °R

V velocity, ft/sec

W mass flow, lb/sec

γ ratio of specific heat at constant pressure to specific heat at constant volume (c_p/c_v)

δ ratio of total pressure to static sea-level pressure (taken at 14.7 lb/sq in.)

η efficiency

θ ratio of total temperature to static sea-level temperature (taken at 518.4° R)

σ ratio of altitude density to sea-level density

Subscripts:

0 - 6 engine stations (fig. 1)

a air

c compressor

g gas

j jet

n net

p propeller

r ram

s shaft

t turbine

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APPENDIX B

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METHOD OF CALCULATING ENGINE PERFORMANCE

The method of calculating engine performance described is general, and may be used for any turbine-propeller engine operating at constant speed with sonic velocity in the turbine-nozzle blading. Conventional compressor and turbine characteristics similar to those shown in figures 6 to 8 and the propeller characteristics typified in figures 3, 9, and 10 must be obtained for the engine investigated.

Operating conditions. - The relation between proportionment of power and the net thrust can best be obtained for constant ram and turbine-inlet-temperature conditions. Under these conditions engine variables upstream of the turbine remain constant. The calculation therefore requires the assumption of an altitude pressure P_0, temperature T_0, density ratio σ, an airplane velocity V_0, and a turbine-inlet temperature T_4.

Fixed engine variables. - The pressure P_2 and the temperature T_2 after ram (the compressor-inlet conditions), were calculated from the following equations:

$$T_2 = T_1 + \frac{V_0^{\,2}}{2gJc_p} \tag{B1}$$

$$P_2 = P_1 \left\{ 1 + \eta_r \left[\left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}} - 1 \right] \right\} \tag{B2}$$

where the engine-inlet conditions at station 1 are assumed equal to the free-stream conditions at station 0.

In equations (B1) and (B2), a constant ram efficiency of 0.90, a specific heat of 0.243, and a ratio of specific heats of 1.395 were assumed.

Corrected compressor speed $N_c/\sqrt{\theta_2}$ was calculated from the relation

$$\frac{N_c}{\sqrt{\theta_2}} = \frac{N_c}{\sqrt{\frac{T_2}{518.4}}} \tag{B3}$$

The corrected engine-air flow $W_a\sqrt{\theta_2}/\delta_2$ was obtained from the compressor characteristics shown in figure 6. From the curve, the corrected air flow can be seen to be a direct function of the known corrected compressor speed. Whenever this characteristic is not exhibited, the engine temperature ratio must be determined before corrected air flow can be obtained.

The actual engine-air flow was obtained from the corrected air flow. By definition,

$$\frac{W_a\sqrt{\theta_2}}{\delta_2} = \frac{W_a\sqrt{\frac{T_2}{518.4}}}{\frac{P_2}{14.7}}$$

Therefore

$$W_a = \frac{W_a\sqrt{\theta_2}}{\delta_2}\left(\frac{\frac{P_2}{14.7}}{\sqrt{\frac{T_2}{518.4}}}\right) \tag{B4}$$

In this relation, 14.7 and 518.4 are the values of sea-level pressure and temperature, respectively.

The engine temperature ratio T_4/T_2 was obtained from the assumed turbine-inlet temperature T_4 and the known compressor-inlet temperature T_2.

The compressor pressure ratio P_3/P_2 was determined from the known values of engine temperature ratio and corrected compressor speed in figure 6.

The compressor temperature ratio T_3/T_2 was obtained from figure 7 for the known values of engine temperature ratio and corrected compressor speed.

The compressor-horsepower parameter $hp_c/(W_aT_4)$ was obtained from the known temperatures and air flow. The compressor horsepower is

$$hp_c = \frac{778}{550} W_a c_p (T_3 - T_2)$$

Dividing by W_aT_4 and simplifying gives

$$\frac{hp_c}{W_aT_4} = \frac{778}{550} \frac{c_p}{T_4} (T_3 - T_2)$$

If a constant value of specific heat equal to 0.243 is assumed and the equation is expressed in terms of known engine variables, the following equation is obtained:

$$\frac{hp_c}{W_aT_4} = 0.344 \frac{\left(\frac{T_3}{T_2} - 1\right)}{\frac{T_4}{T_2}} \tag{B5}$$

Corrected turbine speed was obtained by definition from the equation

$$\frac{N_t}{\sqrt{\theta_2}} = \frac{N_t}{\sqrt{\frac{T_4}{518.4}}} \tag{B6}$$

Division of power. - The assumption of a turbine-horsepower parameter $hp_t/(W_gT_4)$ determined the proportionment of power between the propeller and the jet. The assumed values of the turbine-horsepower parameter were larger than those of the compressor-horsepower parameter and extended to the maximum obtainable for the calculated pressure ratio. Sufficient values of the turbine-horsepower parameter were assumed to determine peak thrust output.

Propeller thrust. - The shaft horsepower hp_s was calculated from the known value of corrected compressor horsepower and the assumed value of corrected turbine horsepower by the following equation:

$$hp_s = W_a T_4 \left(1.02 \frac{hp_t}{W_g T_4} - \frac{hp_c}{W_a T_4}\right) \tag{B7}$$

In equation (B7), the ratio of gas flow W_g to air flow W_a was assumed to equal 1.02.

The horsepower delivered to the propeller hp_p is less than the shaft horsepower by the amount of power absorbed by the reduction gearing and the accessories. The horsepower loss varies only slightly with shaft power at constant engine speed. The following equation was used to obtain horsepower delivered to the propeller:

$$hp_p = 0.995\ hp_s - 77.5 \tag{B8}$$

The propeller characteristics shown in figure 3 are expressed in terms of the propeller power coefficient C_P and the quantity $V_0/(n_p d)$. The propeller power coefficient was evaluated from known engine characteristics by the general equation

$$C_P = \frac{0.0005\ hp_p}{\sigma\left(\frac{N_p}{1000}\right)^3 \left(\frac{d}{10}\right)^5} \tag{B9}$$

For an engine speed of 13,000 rpm, the propeller speeds for the engine investigated were $N_p = 1145$ rpm and $n_p = 1145/60$ rps.

The propeller efficiency η_p was obtained by figure 3 with the calculated values of C_P and $V_0/(n_p d)$.

The thrust output of the propeller F_p was calculated from the equation

$$F_p = \frac{550\ hp_p\ \eta_p}{V_0} \tag{B10}$$

For the analysis of most conditions at sea-level take-off, the calculation of propeller thrust is simplified. Propeller thrust may be obtained directly from figures 9 and 10 for the calculated value of propeller horsepower.

Jet thrust. - The turbine pressure ratio P_5/P_4 was determined by the corrected turbine horsepower and speed. The pressure ratio was obtained from the turbine characteristics at an engine speed of 13,000 rpm shown in figure 8.

The exhaust-nozzle pressure ratio P_5/p_6 was calculated from known pressures in the engine. If a 5-percent pressure loss in the burner is assumed

$$P_4 = 0.95\ P_3$$

Therefore

$$\frac{P_5}{P_0} = 0.95\ \frac{P_3}{P_2}\ \frac{P_5}{P_4}\ \frac{P_2}{P_0} \tag{B11}$$

For subsonic flow in the exhaust nozzle P_5/p_6 is assumed equal to P_5/P_0.

The turbine-outlet temperature T_5 is a function of the turbine-inlet temperature and the turbine power. The following equation presents the relation:

$$T_5 = T_4 \left(1 - \frac{550}{778}\ \frac{1}{c_p}\ \frac{hp_t}{W_g T_4}\right) \tag{B12}$$

This equation was derived from the general expression for horsepower

$$hp_t = \frac{778}{550}\ W_g c_p\ (T_4 - T_5)$$

In equation (B12) the average value of specific heat c_p at a fuel-air ratio of 0.015 and corresponding to the calculated values of P_5/P_4 and T_4 was used (reference 2).

The effective exhaust-nozzle area was calculated from the continuity equation. Evaluated for area this equation is

$$A_e = \frac{RW_g t_6}{V_j p_6} \qquad \text{(B13)}$$

In this equation the jet velocity was evaluated from the expression

$$V_j = \sqrt{2gJ\eta_j c_p T_5 \frac{\left(\frac{P_5}{p_6}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{P_5}{p_6}\right)^{\frac{\gamma-1}{\gamma}}}} \qquad \text{(B14)}$$

and the free-stream temperature at station 6 t_6 was obtained from the expression

$$T_5 - T_6 = \eta_j T_5 \frac{\left(\frac{P_5}{p_6}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{P_5}{p_6}\right)^{\frac{\gamma-1}{\gamma}}} \qquad \text{(B15)}$$

The weight of gas flow W_g is again assumed equal to 1.02 W_a. In evaluating the effective exhaust-nozzle area, P_5/p_6 was assumed to equal P_5/P_0.

The jet thrust was calculated from the equation

$$F_j = \frac{W_g V_j}{g} \qquad \text{(B16)}$$

The values of the weight of gas flow and the jet velocity have previously been calculated and may be substituted in this equation.

Net thrust. - The intake-air drag D for the assumed velocity and operating conditions was calculated from the equation

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$$D = \frac{W_a V_0}{g} \tag{B17}$$

The net-thrust output F_n for the assumed proportioning of power is equal to

$$F_n = F_p + F_j - D \tag{B18}$$

<u>Example of results.</u> - The results obtained from the series of calculations described can be most clearly shown by a plot of engine parameters as a function of the effective exhaust-nozzle area. In this case exhaust-nozzle area is a measure of the proportionment of power. Figure 11 is such a representation of the results obtained with the 10-foot propeller for an altitude of 35,000 feet, a flight velocity of 200 miles per hour, and an engine temperature ratio of 4.5. The curves show the effect of varying exhaust-nozzle area on the propeller horsepower, the jet thrust, the propeller thrust, and the net thrust of the engine.

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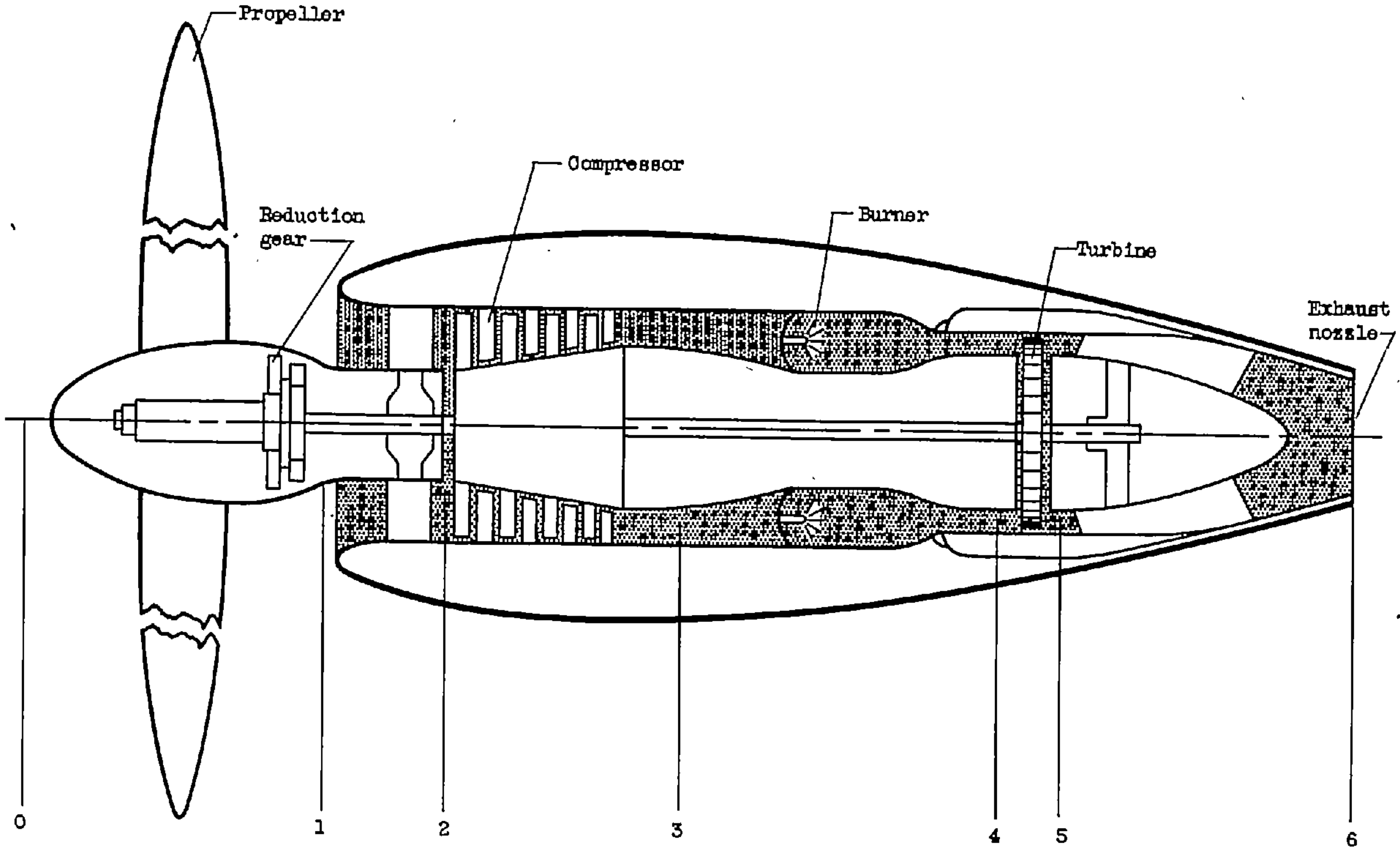

Figure 1. - Schematic diagram of turbine-propeller engine.

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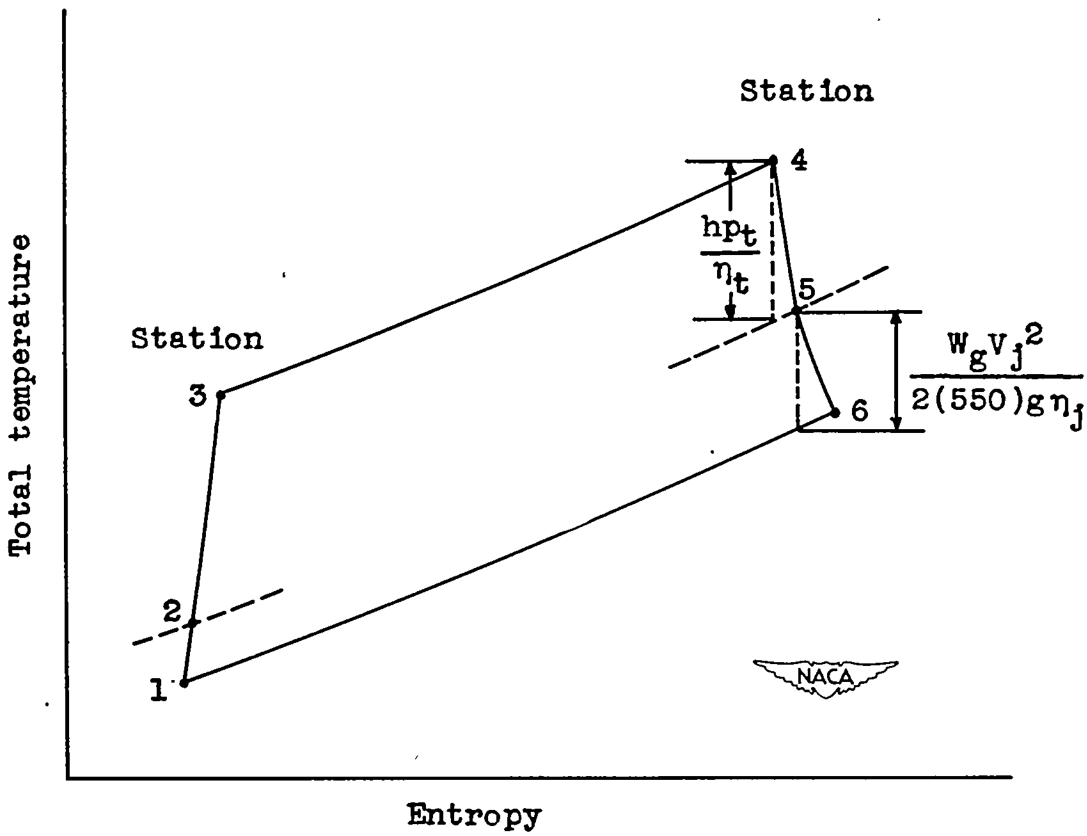


Figure 2. - Temperature-entropy diagram of turbine-propeller-engine cycle.

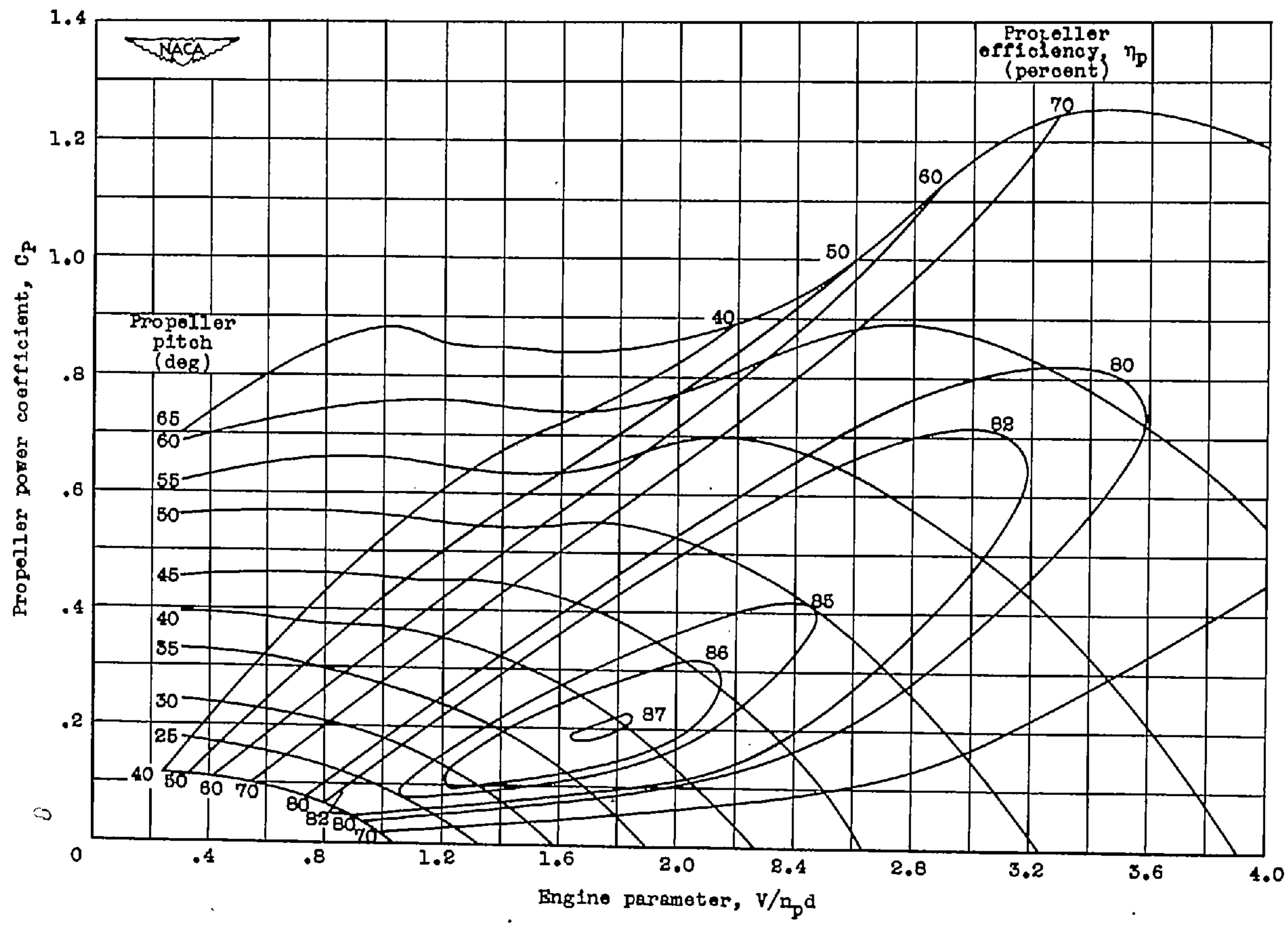
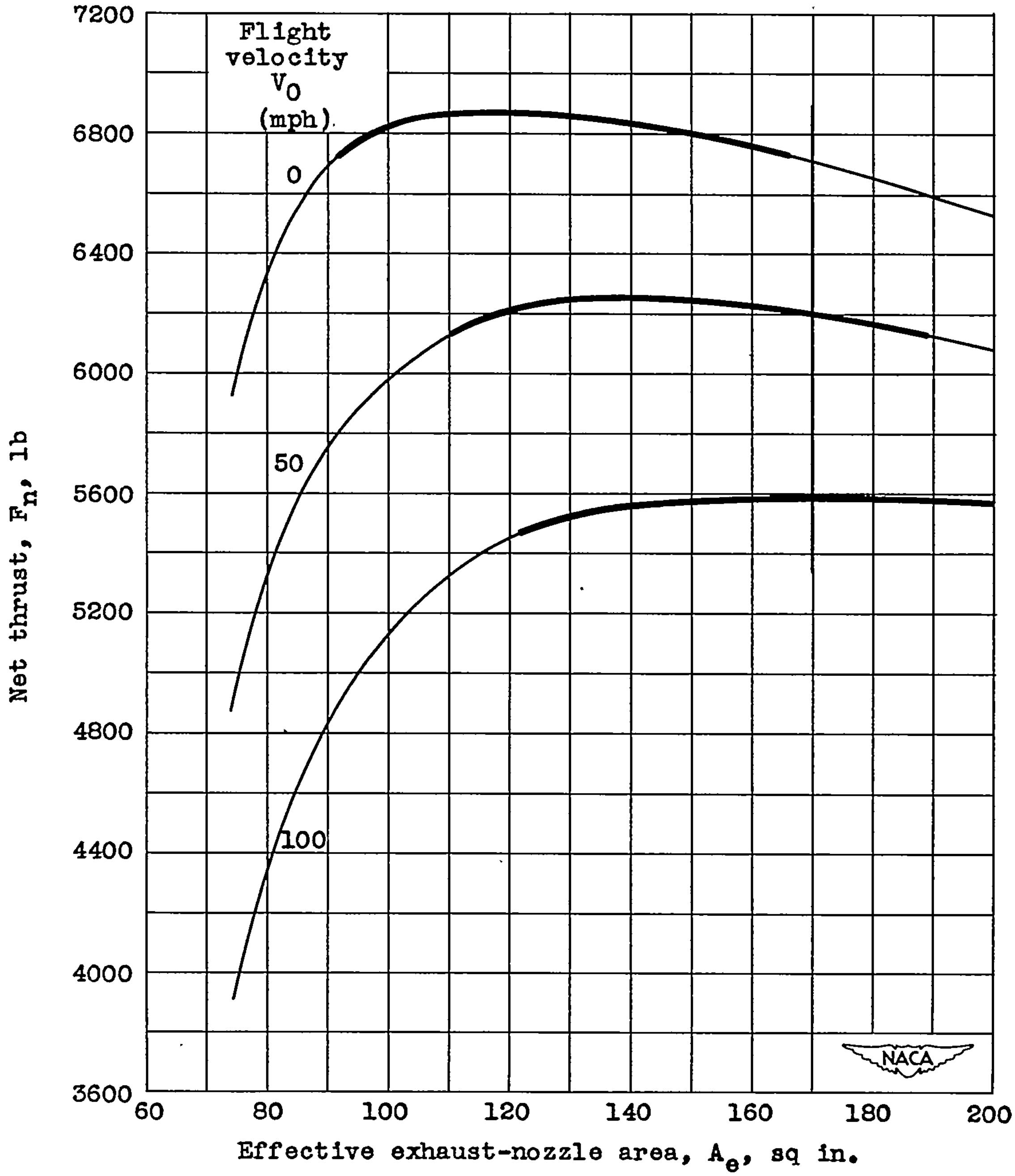
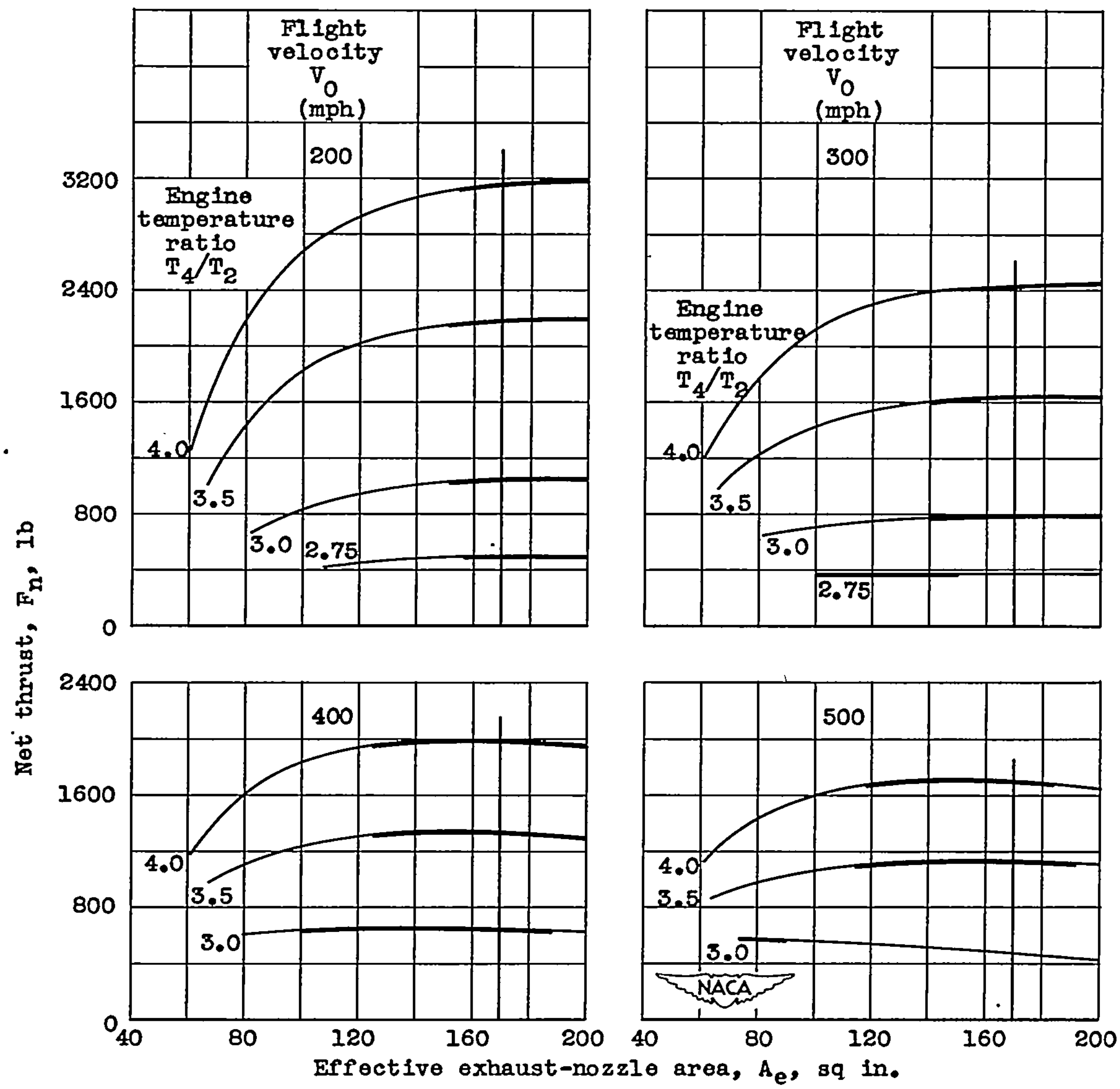


Figure 3. - Characteristics of a typical four-blade single-rotation propeller.



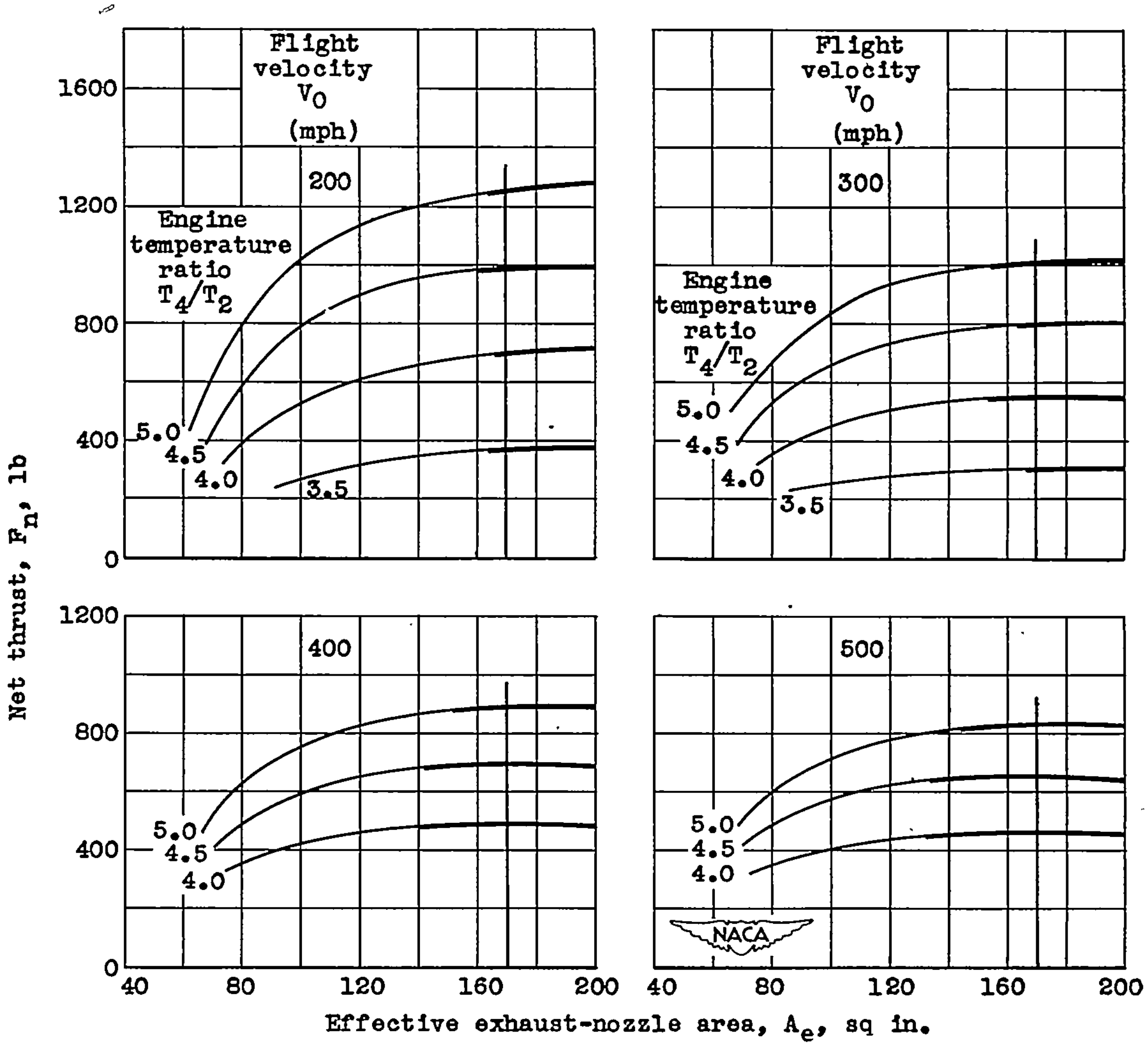
(a) Sea-level take-off; engine temperature ratio, 4.5.

Figure 4. - Effect of power proportionment on net thrust. Heavy-line sections of curves represent thrust within 2 percent of maximum net thrust. Constant-area line of 170 square inches represents best fixed area. Propeller diameter, 13.5 feet.



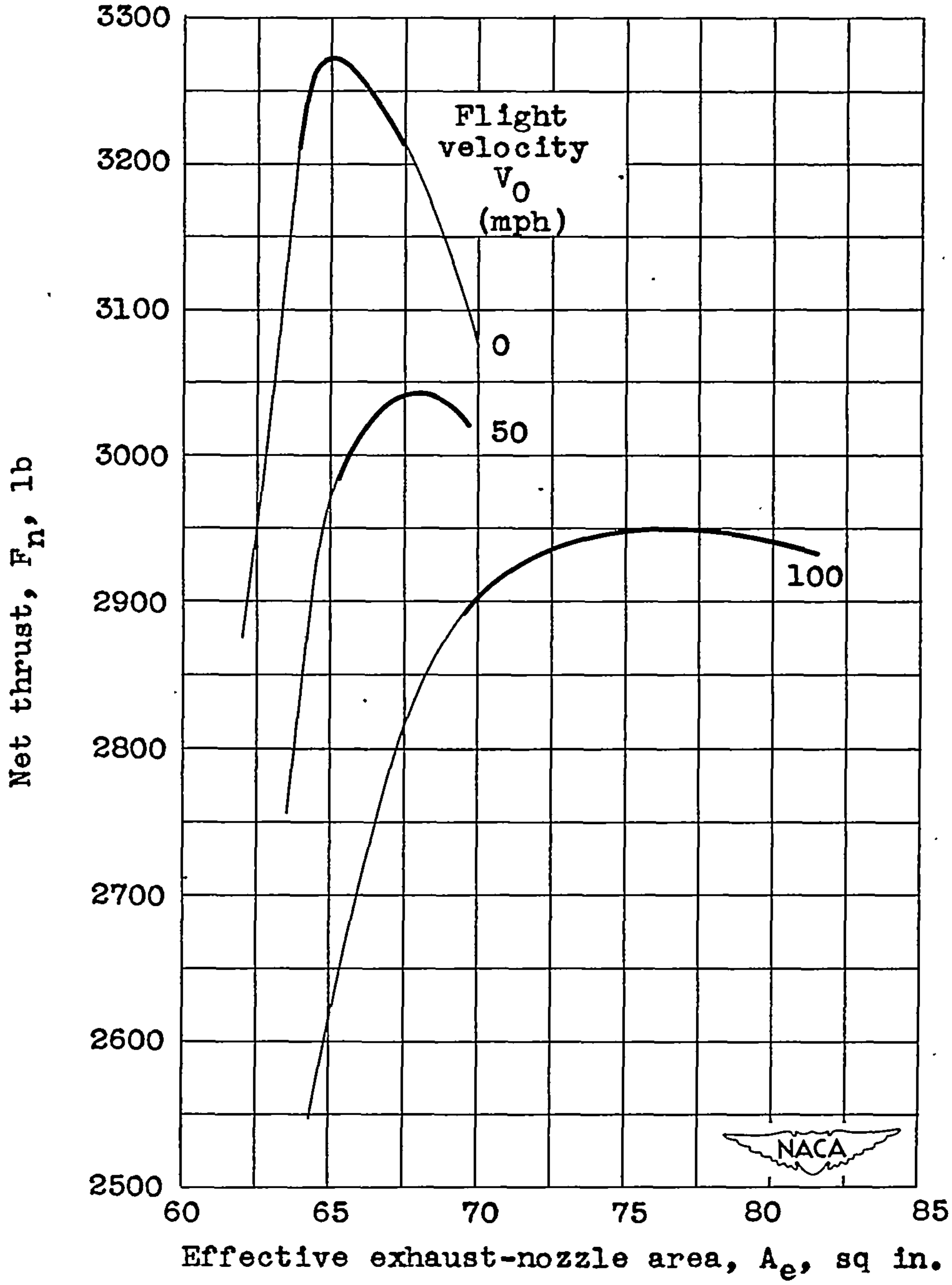
(b) Altitude, sea level.

Figure 4. - Continued. Effect of power proportionment on net thrust. Heavy-line sections of curves represent thrust within 2 percent of maximum net thrust. Constant-area line of 170 square inches represents best fixed area. Propeller diameter, 13.5 feet.



(c) Altitude, 35,000 feet.

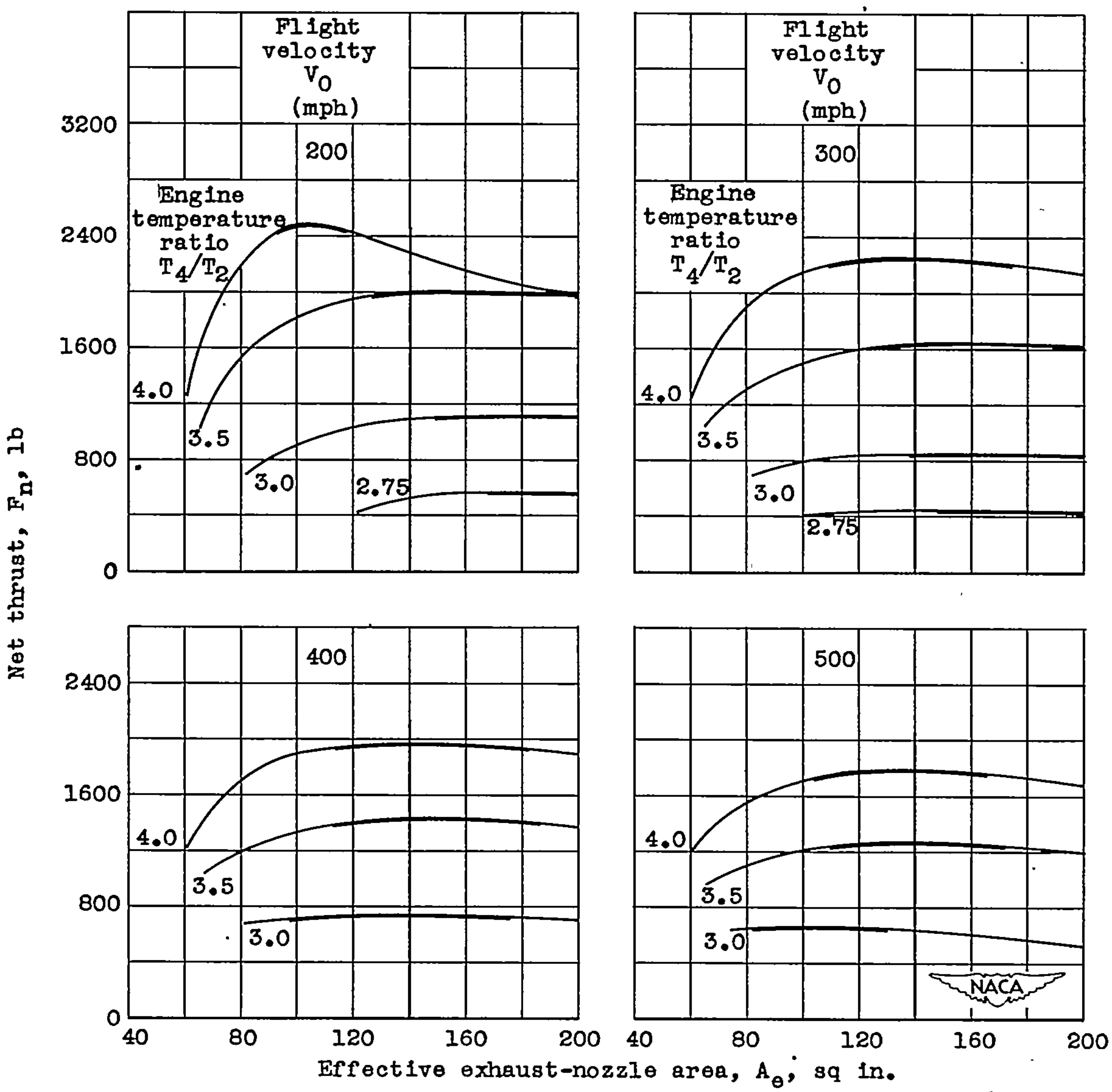
Figure 4. - Concluded. Effect of power proportionment on net thrust. Heavy-line sections of curves represent thrust within 2 percent of maximum net thrust. Constant-area line of 170 square inches represents best fixed area. Propeller diameter, 13.5 feet.



(a) Sea-level take-off; engine temperature ratio, 4.5.

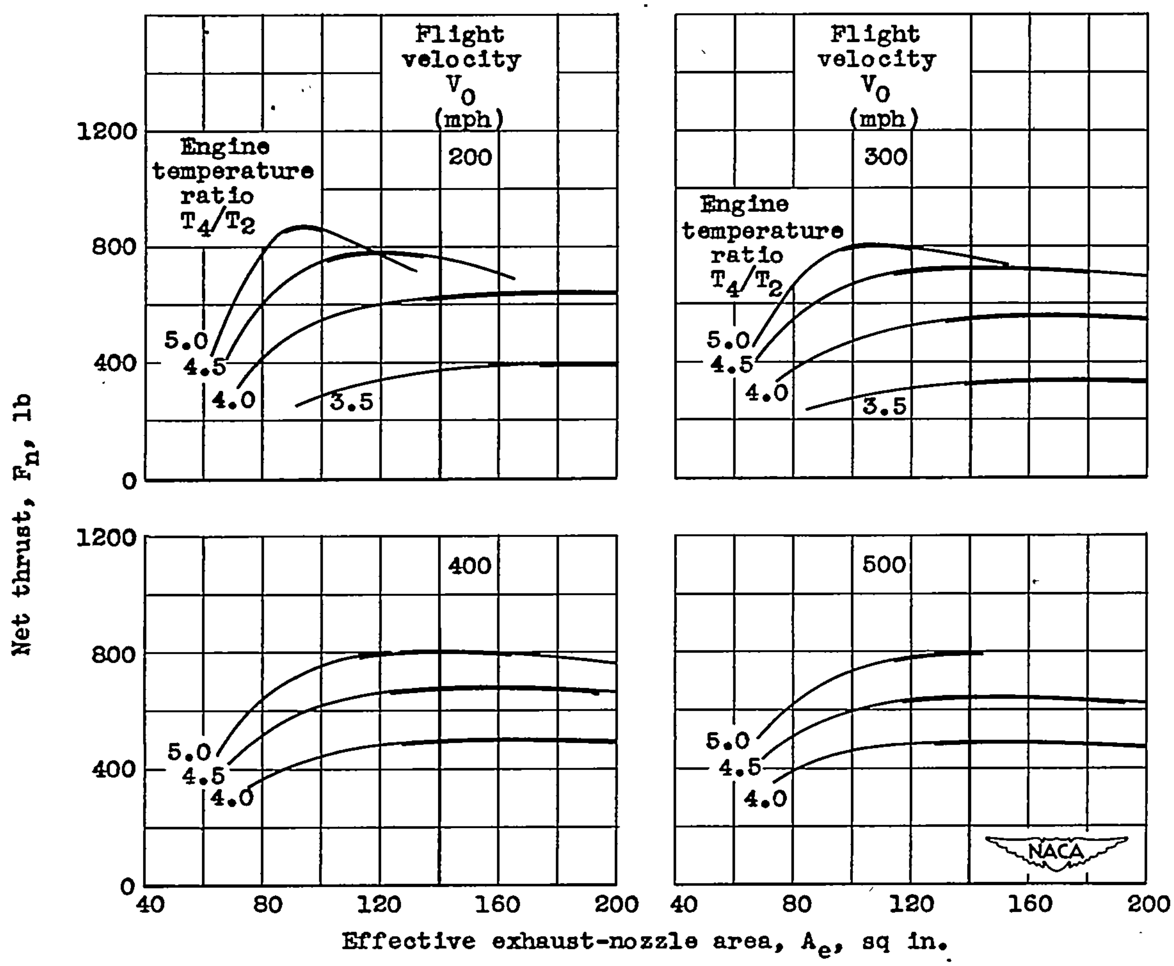
Figure 5. - Effect of power proportionment on net thrust. Heavy-line sections of curves represent thrust within 2 percent of maximum net thrust. Constant-area line of 140 square inches represents best fixed area. Propeller diameter, 10 feet.

1023



(b) Altitude, sea level.

Figure 5. - Continued. Effect of power proportionment on net thrust. Heavy-line sections of curves represent thrust within 2 percent of maximum net thrust. Constant-area line of 140 square inches represents best fixed area. Propeller diameter, 10 feet.



(c) Altitude, 35,000 feet.

Figure 5. - Concluded. Effect of power proportionment on net thrust. Heavy-line sections of curves represent thrust within 2 percent of maximum net thrust. Constant-area line of 140 square inches represents best fixed area. Propeller diameter, 10 feet.

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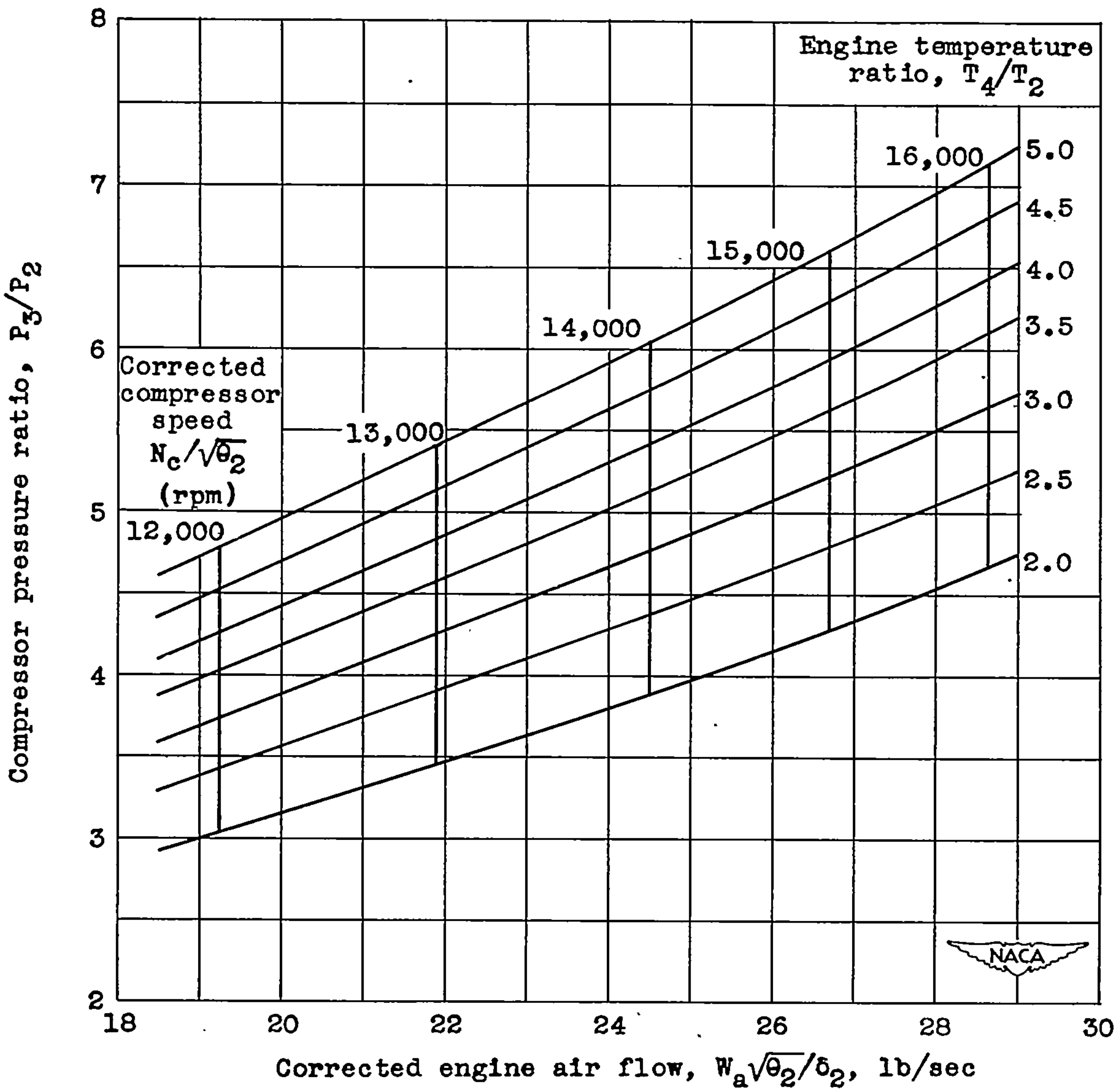

Figure 6. - Compressor performance characteristics.

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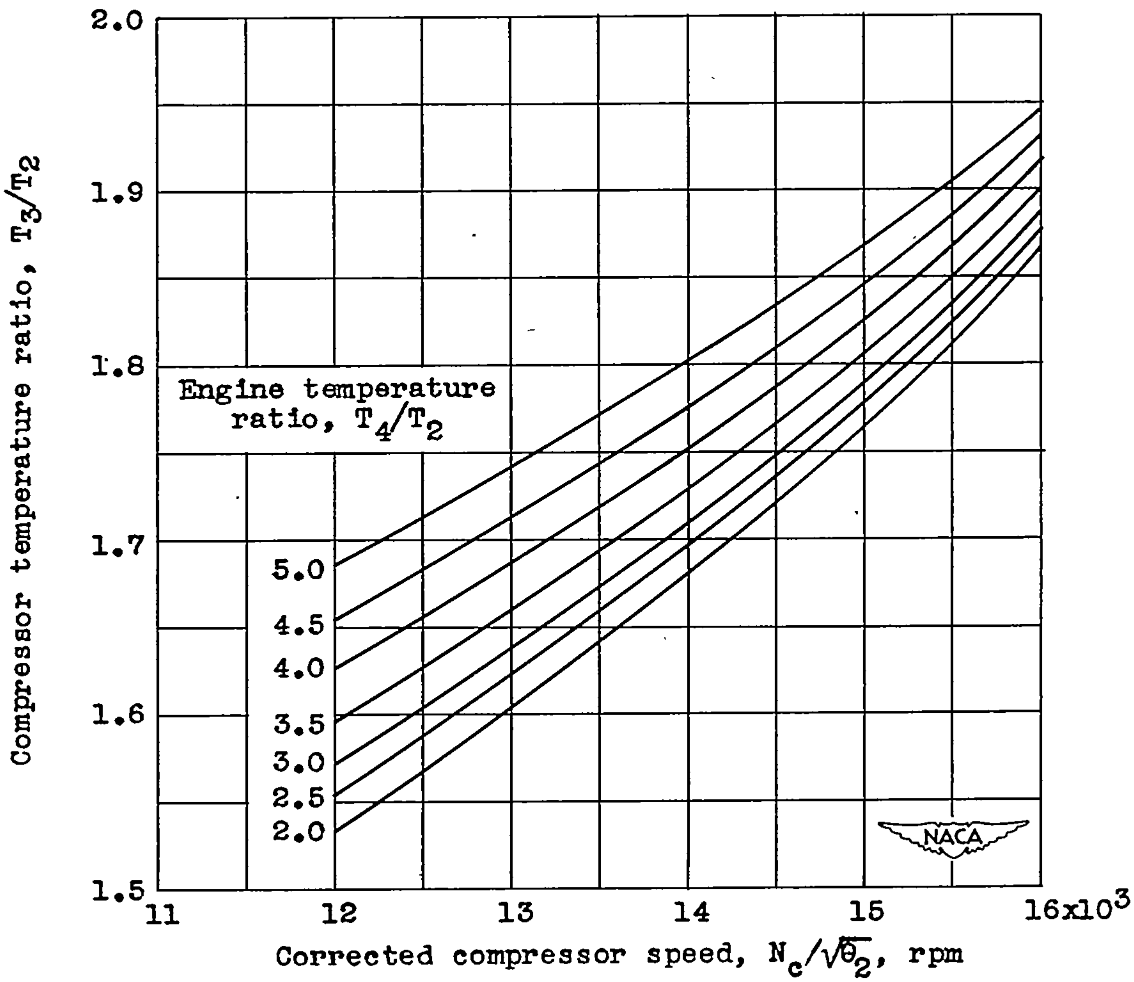

Figure 7. - Compressor temperature-rise characteristics.

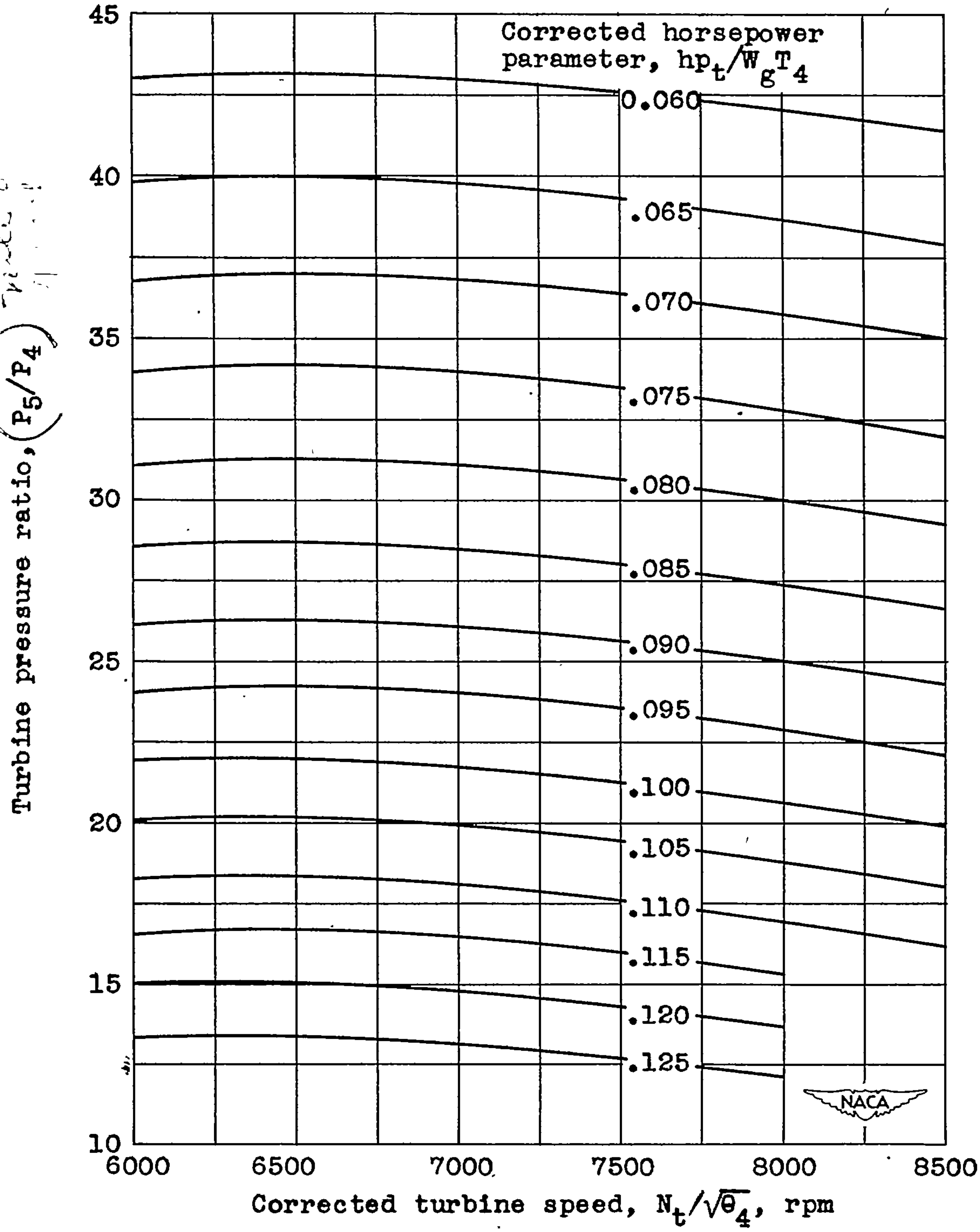

Figure 8. - Turbine performance characteristics at constant engine speed of 13,000 rpm.

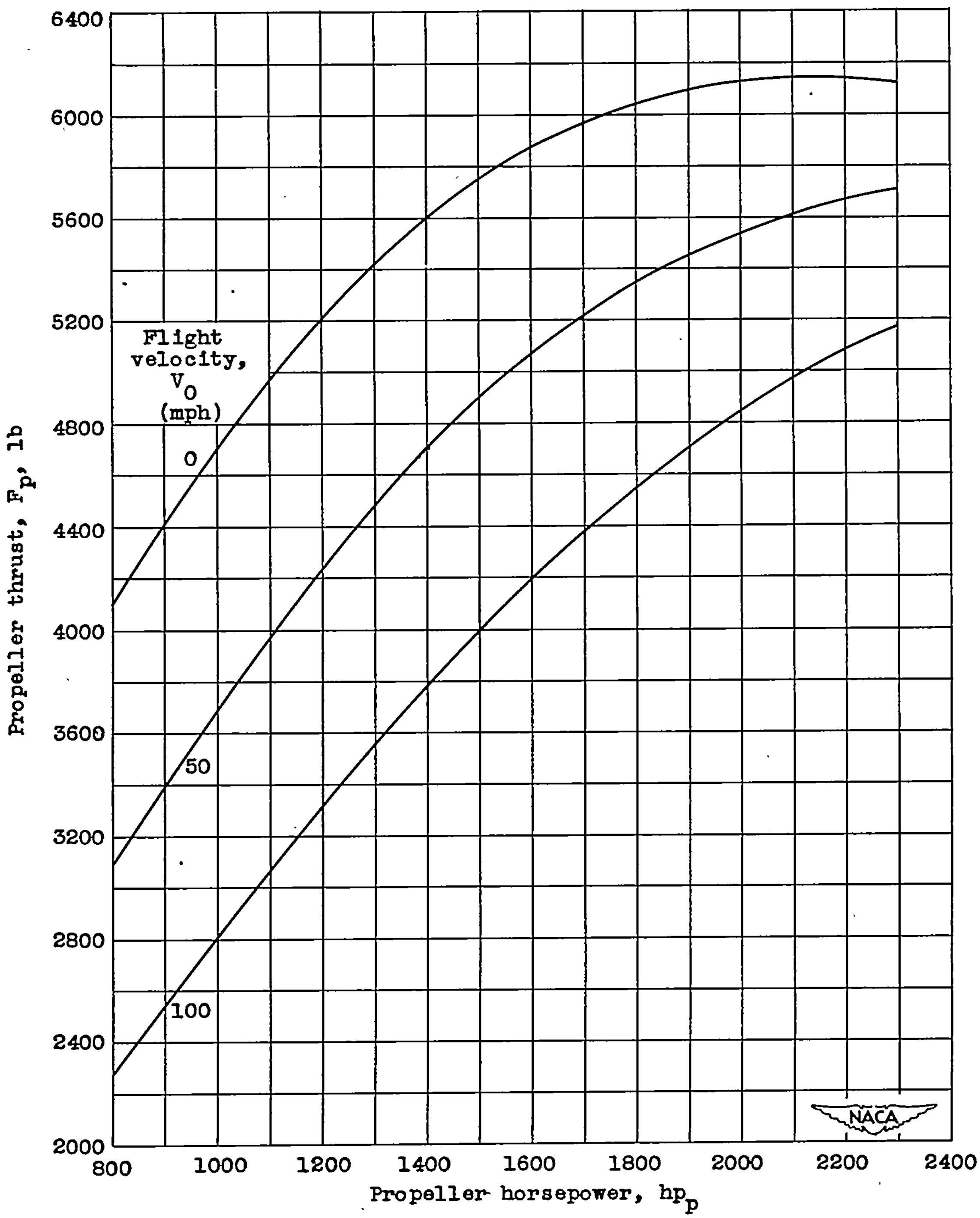

Figure 9. - Take-off characteristics of typical four-blade, 13.5-foot propeller. Propeller speed, 1145 rpm.

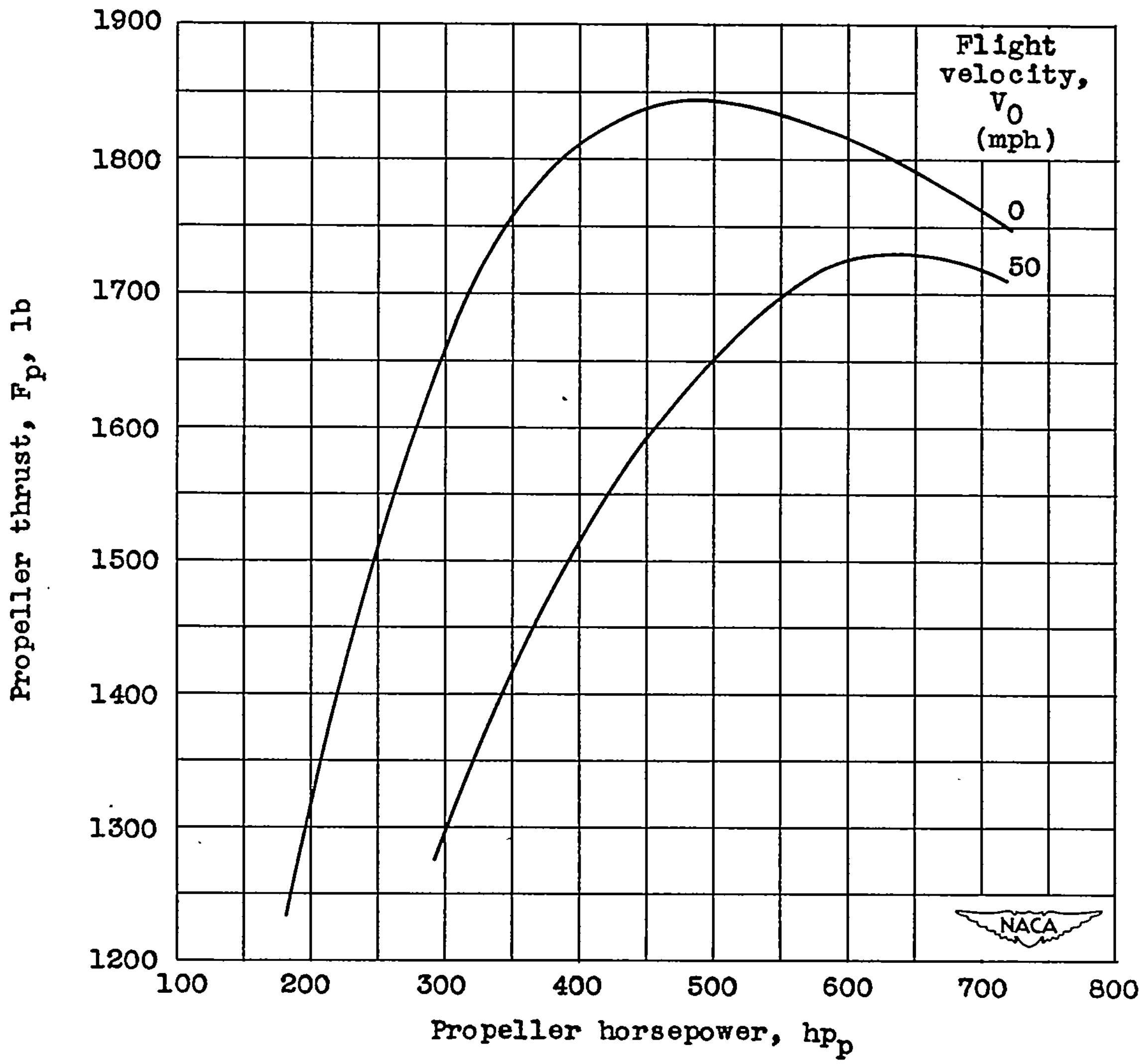

Figure 10. - Take-off characteristics of typical four-blade, 10-foot propeller. Propeller speed, 1145 rpm.

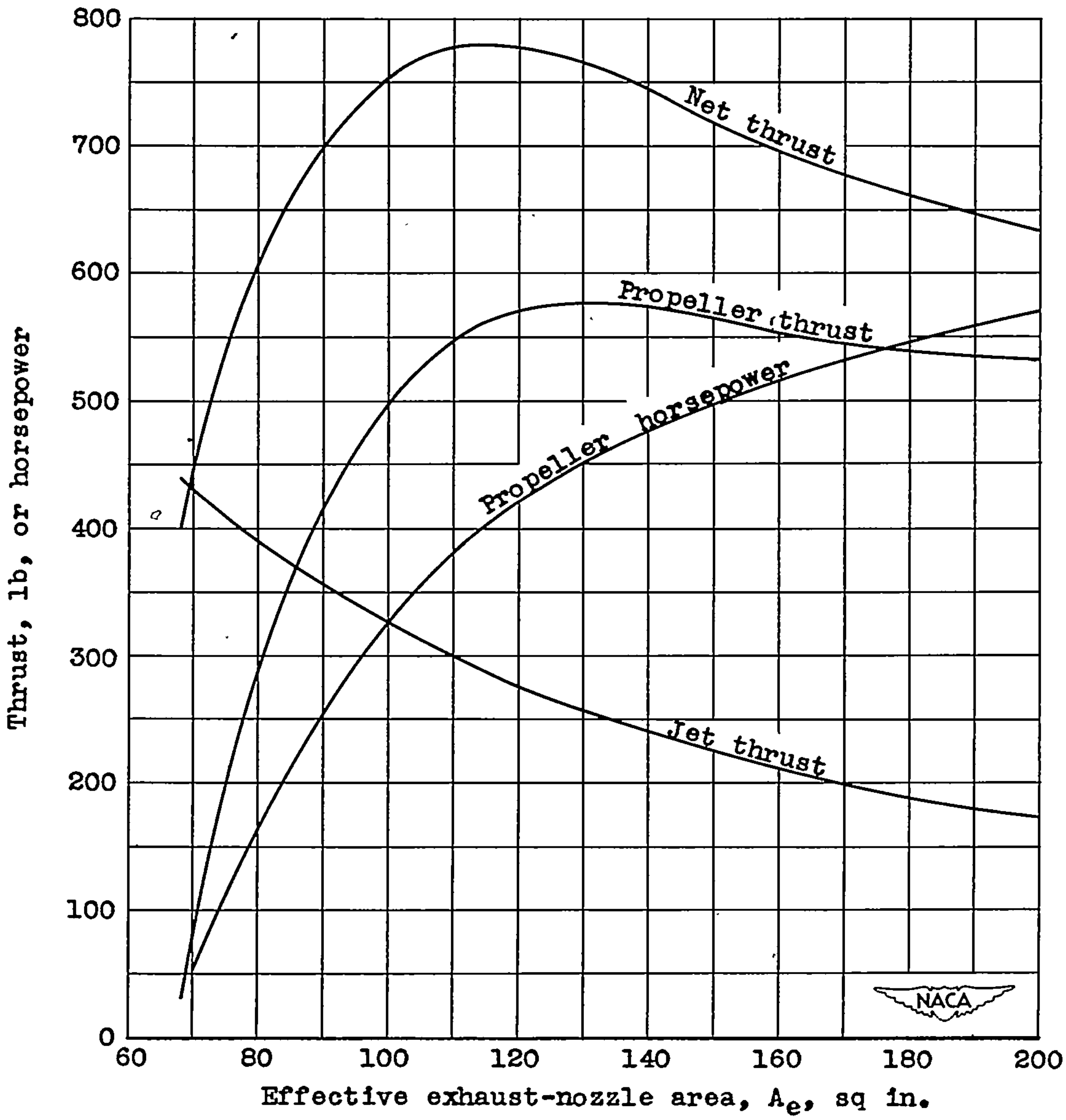

Figure 11. - Effect of varying exhaust-nozzle area on propeller thrust, jet thrust, net thrust, and propeller horsepower. Altitude, 35,000 feet; flight velocity, 200 miles per hour; engine temperature ratio, 4.5; propeller diameter, 10 feet.